The corresponding equations for the in-phase pitching moment about the root quarter-chord line are given by Eq. (1) with $a = 0.82C_{M\alpha}$, $b = -0.085A^2\tan\lambda_{½}$, $d = 4 + A^3/32$, and $h = 0.13C_{M\alpha}$. The out-of-phase pitching moment is given by Eq. (2) with $g = -0.01A\lambda_{½}$, and $f = 6$.

$C_{M\alpha}$ denotes the steady flow pitching moment curve slopes about the wing root quarter-chord point derived from the data sheet of Ref. 7. It is to be noted that the delta wings are excluded from the pitching moment expressions.

Figures 1 and 2 illustrate the quality of the curve fitting obtained. The solid lines represent Eqs. (1) and (2) in comparison with the data plotted as symbols. These figures also indicate the variation of in- and out-of-phase lift and pitching moments with sweep angle.

The general applicability of the curve fitted expressions presented here cannot be defined precisely with confidence. A range of conventional untapered wing planforms with sweep angles between 0 and 45 deg and aspect ratios between 4 and 8 appear to be within the validity of the empirical expressions given. Lift amplitudes for delta wings with small aspect ratios of around 1-2 are also included within the expressions. Some uncertainty does exist for the validity of the expressions for aspect ratios between 2 and 4 although the trends in the lift variations suggest that the likely errors may be small. The use of the pitching moment equations for wing aspect ratio below 3 is not recommended.

It is hoped that these empirical expressions for gust loading will be of some general use for initial calculations and parametric optimization in aircraft design.

References

[1] Rodden, W. P., "State of the Art in Unsteady Aerodynamics," AGARD Report 650, 1976.

[2] McCroskey, W. J., "Some Current Research in Unsteady Fluid Dynamics," *Journal of Fluids Engineering, Transactions of ASME*, Ser. 1, March 1977, pp. 8-39.

[3] Patel, M. H., "Aerodynamic Force on Finite Wings in Oscillatory Flow; An Experimental Study." *AIAA Journal*, Vol. 16, Nov. 1978, pp. 1175-1180.

[4] Patel, M. H., "On Delta Wings in Oscillatory Flow," *AIAA Journal*, Vol. 18, May 1980, pp. 481-486.

[5] Patel, M. H. and Hancock, G. J., "A Gust Tunnel Facility," Aeronautical Research Council, R&M 3802, 1977.

[6] Patel, M. H., "On the Linear Superposition of Aerodynamic Forces on Wings in Periodic Gusts," *Aeronautical Journal*, Vol. 82, June 1978, pp. 267-272.

[7] "Lift Curve Slope and Aerodynamic Centre Position of Wings in Inviscid Subsonic Flow," Engineering Sciences Data Unit, 251 Regent Street, London, England, E.S.D.U. Item 70011, 1978.

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Wake Rollup and the Kutta Condition for Airfoils Oscillating at High Frequency

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Introduction

ONE of the basic assumptions of airfoil theory deals with the presence of a stagnation point at the (usually) sharp trailing edge. This is commonly called the Kutta (or Kutta-Joukovsky) condition. While the existence of this condition is well established for steady, nonseparated flow situations, its validity for time-dependent cases is still controversial.[1-5] The Kutta condition is applied almost axiomatically in most numerical studies of unsteady wing theory,[6] but conclusions of experimentalists seem to differ, especially for higher values of the reduced frequency.

In the present Note, we show that the Kutta condition can be applied for engineering force and moment prediction in unsteady small amplitude nonseparated flows, even when the reduced frequency is $\sigma \gg 1$. The excellent agreement between these calculations of downstream wake rollup and some smoke flow visualizations[7] indicates that the assumption of the Kutta condition is at least macroscopically correct and will suffice for integral force calculations.

Published experimental pressure data,[1-5] generally, showed good agreement between measured values and those predicted by linearized wing theory; the only deviations found were in the vicinity of the trailing edge (TE). Conclusions drawn from these data, however, differ considerably. Satyanarayana and Davis[3] concluded from their pressure measurements that the Kutta condition is not valid for reduced frequencies of $\sigma \geq 0.6$. Furthermore, Kadlec and Davis[4] argued that above this frequency, wake rollup behind oscillating airfoils is large and therefore mathematical models based on small disturbance theory are not applicable. The data of Ref. 3, however, show excellent agreement of measured and predicted pressures at 40% chord, up to their highest tested frequency ($\sigma = 1.23$). Pressure measurements by Fleeter,[5] both on isolated and a cascade of flat plates, showed similar trends. On the basis of data taken at 92 and 96% of the chord, he concluded that the pressure difference at the trailing edge would be zero throughout his test conditions ($\sigma \leq 7.5$, $\alpha < 10$ deg).

Further experimental evidence in favor of the existence of the Kutta condition up to reduced frequencies of $\sigma = 3.9$ were obtained[1] by placing the airfoil in the vortex street shed from a cylinder. This frequency range was further extended by Archibald[2] up to $\sigma = 50$. In these papers slight deviations from linear theory pressure predictions were observed at the vicinity of the TE above $\sigma = 0.6$.

Calculation of Wake Rollup

The aforementioned experimental results[1-5] show that for small TE amplitudes and very high frequencies the integral quantities such as lift are consistent with results of linearized theory. On the other hand, modified linear theory wake rollup calculations[9] are in excellent agreement when compared with flow visualization data. Furthermore, it was shown that for a given frequency, TE amplitude, and airfoil lift (circulation) history, the wake rollup is unique and the effect of this wake rollup on the lift is minimal.

In this section we present the results of a modified linear airfoil theory that shows that both lift and wake rollup of oscillating airfoils can be predicted for higher frequencies, with the aid of the Kutta condition. It is assumed that the airfoil vertical displacement $z = h(x,t)$ due to its periodic heaving and/or pitch motions is such that $h(x,t)/c \ll 1$ where c is the chord length. The TE vertical amplitude (a) is small so that no major separations occur. This limitation is a function of the Reynolds number and reduced frequency (σ) but here we assume that $a/c < 0.1$.

The wake rollup calculation is performed such that at each time interval Δt a vortex element is shed from the TE, (i.e., the circles in Fig. 1). The strength of each vortex shed γ_i is equal in magnitude and opposite in sign to the change in the circulation of the airfoil bound circulation Γ_f, during that time step. The vortex wake rollup is obtained by moving the wake elements by displacements $\Delta x_i(t)$, $\Delta z_i(t)$ such that

$$\begin{pmatrix} \Delta x_i(t) \\ \Delta z_i(t) \end{pmatrix} = \begin{pmatrix} u_i(t) \\ w_i(t) \end{pmatrix} \Delta t \qquad (1)$$

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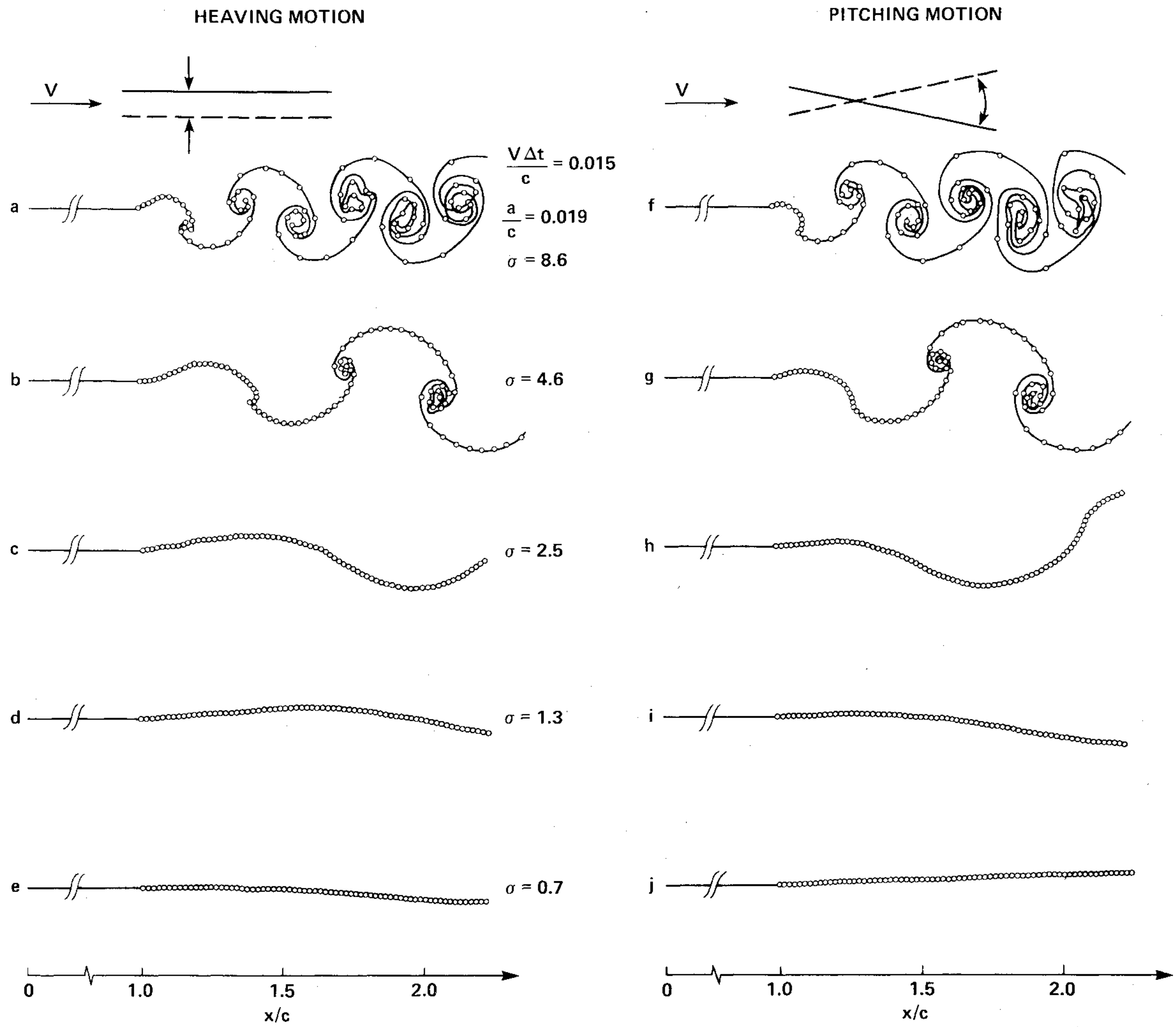

Fig. 1 Wake rollup behind heaving and pitching flat plates, ($\sigma = \omega c/2V$; reduced frequency, Δt: numerical time step).

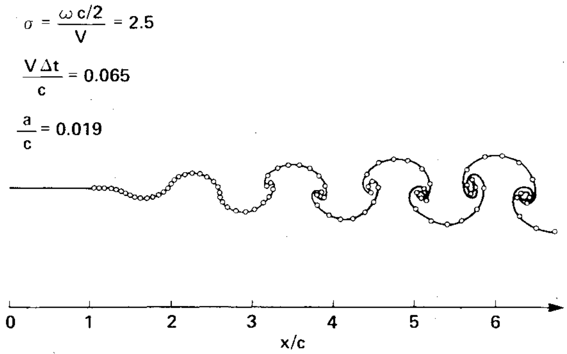

Fig. 2 Far wake development of case c in Fig. 1.

where u_i and w_i are the wake-airfoil induced velocities measured at each vortex location. Once the wake rollup procedure is completed for a given time step, the airfoil is advanced along its trajectory and the procedure is repeated. Details of this calculation appear in Ref. 6.

The various calculated wake rollup patterns behind oscillating airfoils are shown in Fig. 1. The maximum trailing-edge displacement was set constant ($a/c = 0.019$) as in Ref. 7, for both the heaving motion (Fig. 1, a-e) and for the foil pitching motion about its quarter chord (Fig. 1, f-j). The reduced frequency σ was varied through a wide range to correspond to values used in the smoke visualizations.[7] Recent Schlieren flow visualizations shown by Ref. 4 are consistent with the calculated wake shapes on Fig. 1. These measurements were conducted in the vicinity of the trailing edge only; as a result they[4] concluded that at about $\sigma = 3.28$ and $a/c = 0.04$ the wake deforms to a sinusoidal shape but does not break up into individual vortex agglomerations. For this purpose Fig. 1c was extended at Fig. 2 to show the far wake behind the airfoil; to indicate that vortex breakup does occur at a larger distance. Evidence to this phenomenon is also found in Bratt's[7] report. At higher reduced frequencies ($\sigma > 1$) the downwash on the airfoil surface is mainly a component of its oscillatory motion and wake rollup influence is small. Therefore, the argument that wake distortion might affect analytical predictions based on potential theory as stated in Ref. 4 is probably not applicable here. Moreover, in high-frequency motions the contribution of the potential time derivative $\partial\phi/\partial t$ to the lift becomes more important, i.e., force due to the acceleration of the surrounding fluid is considerably increased relative to the far wake influence.

Comparison of Figs. 1c and 2 gives an indication of consistency of the calculational scheme. Even with the four times larger time step used in Fig. 2 the predicted wake shape was not affected. In addition, it is shown that vortex sheet breakup is likely to occur at reduced frequencies $\sigma > 2$.

The important role of trailing-edge displacement parameter a/c on the shape of the wake rollup is demonstrated when comparing figures a-e to f-j in Fig. 1. The wake shapes are very similar although the two oscillatory modes considered are completely different.

Concluding Remarks

Wake rollup calculations, based on the Kutta condition, showed good agreement with available flow visualization data. It is concluded therefore that when TE displacement is small ($a/c<0.1$), the range of linearized theory calculations using the Kutta condition can be extended far beyond reduced frequencies of $\sigma>1$. There is also a need for an extensive experimental study of airfoil lift and wakes, over a wide range of Reynolds number, frequency, and TE amplitudes. When comparing these arguments to those of steady airfoil theory, it is noticeable that in steady flow under slight TE separation the airfoil lift is almost unaffected.[8] Similar reasons, probably, lead to the local pressure violation (compared to linearized theory predictions) reported[1-3] near the TE resulting in some phase shift. Measured pressures in the front section of the airfoil that provide the major contribution to the lift, however, were in agreement with linear predictions.

The effect of wake rollup on the calculated lift of the cases shown was small since the vertical velocities at the higher frequencies are mainly a result of the airfoil motion, and wake induced velocities are much smaller. Furthermore, both calculated and visualized data show that breakup occurs when $\sigma>2$.

References

[1] Commerford, G.L. and Carta, F.O., "Unsteady Aerodynamic Response of a Two-Dimensional Airfoil at High Reduced Frequency," *AIAA Journal,* Vol. 12, Jan. 1974, pp. 43-48.

[2] Archibald, F.S., "Unsteady Kutta Condition at High Values of the Reduced Frequency Parameter," *Journal of Aircraft,* Vol. 12, June 1965, pp. 545-550.

[3] Satyanarayana, B. and Davis, S., "Experimental Studies of Unsteady Trailing-Edge Conditions," *AIAA Journal,* Vol. 16, Feb. 1978, pp. 125-129.

[4] Kadlec, R.A. and Davis, S., "Visualization of Quasiperiodic Flows," *AIAA Journal,* Vol. 17, Nov. 1979, pp. 1164-1169.

[5] Fleeter, S., "Trailing Edge Condition for Unsteady Flows at High Reduced Frequency," AIAA Paper 79-0152, Jan. 1979.

[6] Katz, J. and Weihs, D., "Hydrodynamic Propulsion by Large Amplitude Oscillation of an Airfoil with Chordwise Flexibility," *Journal of Fluid Mechanics,* Vol. 88, Pt. 3, 1978, pp. 485-497.

[7] Bratt, J.B., "Flow Pattern in the Wake of an Oscillating Airfoil," Aeronautical Research Council, R&M 2773, 1953.

[8] Abbott, I.H. and Doenhoff, A.E., *Theory of Wing Sections,* Dover, N.Y., 1959, pp. 528-529.

[9] Katz, J., "Study of the Forces due to Large Amplitude Lateral Motions of a Flexible Wing," D.Sc. Thesis, Technion, I.I.T., Haifa, Israel, May 1977.

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Asymptotic Suction Flow near a Corner

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I. Introduction

SUCTION has been used for boundary-layer control to increase lift and reduce the drag of airfoils. A surprisingly simple case can be obtained when the velocity components are independent of longitudinal coordinate. In this case, Schlichting[1] reported the case of a flat plate at zero incidence with uniform suction. Several unsteady cases were published in Refs. 2 and 3. The flow of non-Newtonian power-law fluid along a flat plate with uniform suction is considered in Ref. 4. Recently, Zierep[5] published a solution of the Rayleigh-Stokes problem near the corner formed by two perpendicular flat plates. The interference near the corner can be observed. Liu and Ismail[6] solved the case of asymptotic suction flow of natural convection near a corner formed by two perpendicular flat plates embedded in a porous medium. Both temperature and velocity profiles were obtained theoretically. This Note is intended to present an exact solution to the Navier-Stokes equations of the flow of incompressible fluid near a corner when the asymptotic suction condition is reached. The velocity profile is obtained and the interaction of the two plates can be observed.

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II. Formulation and Solution

We consider the steady flow of a viscous incompressible fluid near a corner formed by two perpendicular flate plates at zero incidence with uniform suction. At large distances from the leading edge the asymptotic suction condition can be reached and the velocity profile is independent of the longitudinal distance.

Liu and Ismail[6] demonstrated that, under this condition, the velocity components normal to the plates are both constants and equal to b throughout the flowfield in the asymptotic region. Then the Navier-Stokes equations reduce to

$$-b\frac{\partial u}{\partial y}-b\frac{\partial u}{\partial z}=\nu\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} \tag{1}$$

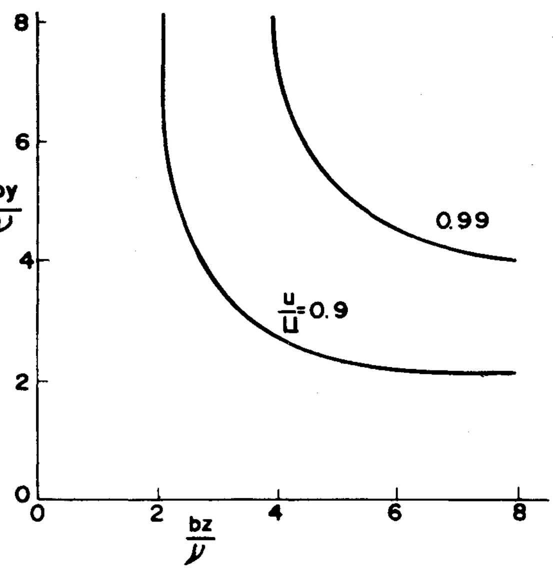

Fig. 1 Constant velocity contours.

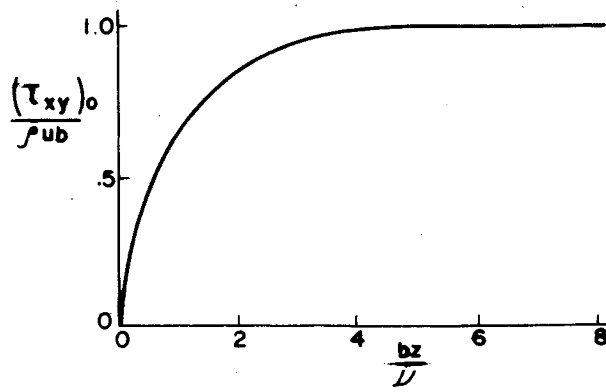

Fig. 2 Variation of the wall shearing stress from the intersection line.